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Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Optimal Cruise Performance

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Nomenclature

a = speed of sound
C_{D_0} = zero-lift drag coefficient
C'_{D_0} = derivative of the zero-lift drag coefficient with respect to Mach number
C_L = lift coefficient
c = specific fuel consumption
D = aerodynamic drag
h = altitude
K = induced drag coefficient
K' = derivative of the induced drag coefficient with respect to Mach number
M = Mach number
q = dynamic pressure
R = total downrange
S = reference aerodynamic area
T = thrust
V = speed
W = weight
W_f = final weight
W_i = initial weight
x = downrange
$(\ )_h$ = ( ) evaluated at constant h
$(\ )_M$ = ( ) evaluated at constant M
$(\ )_V$ = ( ) evaluated at constant V
$(\ )^*$ = optimal value of ( )

Introduction

OPTIMAL cruise performance has received considerable attention from both theoretical and numerical analysts. The first group, Anderson,[1] Nicolai,[2] Vinh,[3] and Perkins and Hage,[4] has determined the condition for best cruise speed. The second group made up of engineers that do trajectory analysis by numerically integrating the equations of motion, has determined best cruise speed and altitude by maximizing n.mi./lb, $V/\dot{W}$. Miller[5] has determined best climb speed and altitude for cruise climb.

The purpose of this effort is to determine the optimal cruise conditions for constant cruise altitude and cruise climb.

Formulation

The relation for cruise range is

$$R = \int_{W_f}^{W_i} \frac{V}{cD}\,\mathrm{d}W \tag{1}$$

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The drag coefficient is based upon a parabolic polar

$$C_D = C_{D_0} + KC_L^2 \tag{2}$$

The drag coefficients are constant until the drag rise region is reached; the coefficients are Mach number dependent thereafter.

Cruise Climb

Optimization will first be with respect to speed and then with respect to altitude. Assume that the specific fuel consumption is minimum and a function of Mach number and altitude

$$c = c(M, h) \tag{3}$$

For maximum range, the speed is selected that maximizes V/cD. The derivative of the logarithm of V/cD with respect to speed is

$$\frac{1}{V} - \frac{1}{c}\left(\frac{\partial c}{\partial M}\right)_h \left(\frac{\partial M}{\partial V}\right)_h - \frac{1}{D}\left(\frac{\partial D}{\partial V}\right)_h = 0 \tag{4}$$

The drag is

$$D = qSC_D \tag{5}$$

Since the lift equals the weight, the drag becomes

$$D = qSC_{D_0} + \frac{KW^2}{qS} \tag{6}$$

The derivative of the drag is

$$\left(\frac{\partial D}{\partial V}\right)_h = \left(\frac{\partial D}{\partial q}\right)_h \left(\frac{\partial q}{\partial V}\right)_h + \left(\frac{\partial D}{\partial M}\right)_h \left(\frac{\partial M}{\partial V}\right)_h \tag{7}$$

where

$$\left(\frac{\partial D}{\partial q}\right)_h = C_{D_0}S - \frac{KW^2}{q^2S} \tag{8}$$

$$\left(\frac{\partial q}{\partial V}\right)_h = \frac{2q}{V} \tag{9}$$

$$\left(\frac{\partial D}{\partial M}\right)_h = qSC'_{D_0} + \frac{K'W^2}{qS} \tag{10}$$

$$\left(\frac{\partial M}{\partial V}\right)_h = \frac{1}{a} \tag{11}$$

Equation (4) becomes

$$\frac{1}{V} - \frac{1}{ac}\left(\frac{\partial c}{\partial M}\right)_h - \frac{1}{D}\left[\frac{2}{V}\left(qC_{D_0}S - \frac{KW^2}{qS}\right) + \frac{1}{a}\left(qSC'_{D_0} + \frac{K'W^2}{qS}\right)\right] = 0 \tag{12}$$

Multiplying by DV and substituting for D gives

$$-qSC_{D0} + 3\frac{KW^2}{qS} - M\left(qSC'_{D0} + \frac{K'W^2}{qS}\right) - \frac{MD}{c}\left(\frac{\partial c}{\partial M}\right)_h = 0 \tag{13}$$

Below the drag rise, the zero-lift drag and induced drag coefficients are constant. Equation (13) reduces to the known results for constant specific fuel consumption

$$C_{D0} = 3KC_L^2 \tag{14}$$

$$D = \tfrac{4}{3}W(3KC_{D0})^{1/2} \tag{15}$$

$$V^* = \left(\frac{2W}{\rho S}\right)^{1/2}\left(\frac{3K}{C_{D0}}\right)^{1/4} \tag{16}$$

$$\max\frac{V}{cD} = \frac{1}{4c}\left(\frac{2}{W\rho S}\right)^{1/2}\left(\frac{27}{KC_{D0}^3}\right)^{1/4} \tag{17}$$

From Eqs. (16) and (17), it can be observed that V and V/cD both increase with increasing altitude. This obviously cannot go on indefinitely. Eventually the thrust exceeds the drag or the drag is in the drag rise region. For the former, the global optimal altitude is obtained and

$$D = \max T(V^*, h) \tag{18}$$

When the speed is greater than the speed for the onset of drag rise, then the derivatives of the zero-lift drag and induced drag coefficients come into play. Then the additional terms in Eq. (13) must be included. In the drag rise region, the optimal cruise altitude requires optimization of $1/cD$ with respect to altitude.

Optimization of $1/cD$ with respect to altitude results in the following equation:

$$\frac{1}{D}\left(\frac{\partial D}{\partial h}\right)_V + \frac{1}{c}\left(\frac{\partial c}{\partial h}\right)_V = 0 \tag{19}$$

where

$$\left(\frac{\partial D}{\partial h}\right)_V = \left(\frac{\partial D}{\partial q}\right)_V\left(\frac{\partial q}{\partial h}\right)_V + \left(\frac{\partial D}{\partial M}\right)_V\left(\frac{\partial M}{\partial h}\right)_V \tag{20}$$

$$\left(\frac{\partial q}{\partial h}\right)_V = \frac{q}{\rho}\frac{d\rho}{dh} \tag{21}$$

$$\left(\frac{\partial M}{\partial h}\right)_V = -\frac{M}{a}\frac{da}{dh} \tag{22}$$

Equation (19) becomes

$$\left(qSC_{D0} - \frac{KW^2}{qS}\right)\frac{1}{\rho}\frac{d\rho}{dh} - \left(qSC'_{D0} + \frac{K'W^2}{qS}\right)\frac{M}{a}\frac{da}{dh} + \frac{D}{c}\left(\frac{\partial c}{\partial h}\right)_V = 0 \tag{23}$$

Special Case: Stratospheric Flight and Constant c

In the stratosphere, the speed of sound is constant. If the specific fuel consumption is also constant, then Eqs. (13) and (23) reduce to the following for cruise climb:

$$-qSC_{D0} + 3\frac{KW^2}{qS} - M\left(qSC'_{D0} + \frac{K'W^2}{qS}\right) = 0 \tag{24}$$

$$qSC_{D0} - \frac{KW^2}{qS} = 0 \tag{25}$$

The solution for optimal dynamic pressure is obtained from Eq. (25)

$$q = \frac{W}{S}\left(\frac{K}{C_{D0}}\right)^{1/2} \tag{26}$$

Substitution into Eq. (24) gives

$$2 - M\left(\frac{C'_{D0}}{C_{D0}} + \frac{K'}{K}\right) = 0 \tag{27}$$

It can be easily shown that this is the same as the maximum with respect to Mach number of the following product:

$$\left[M\left(\frac{L}{D}\right)_{\max}\right] \tag{28}$$

Since

$$\left(\frac{L}{D}\right)_{\max} = \frac{1}{2(KC_{D0})^{1/2}} \tag{29}$$

taking the derivative of the product in relation (28) gives

$$2 - M\left(\frac{C'_{D0}}{C_{D0}} + \frac{K'}{K}\right) = 0 \tag{30}$$

Since the drag coefficients are constant for Mach numbers in the subsonic range, the maximum of the product of the Mach number and maximum lift-to-drag ratio occurs in the subsonic drag rise region. Therefore, the optimal cruise Mach number is not at values less than where the drag rise occurs.

The procedure for determining optimal cruise Mach number and altitude is straightforward. First compute maximum lift-to-drag ratio as a function of Mach number. Then plot the product of this ratio and the Mach number. Select the Mach number that maximizes this product. Assume that the trajectory is in the stratosphere. Compute the optimal cruise speed and the optimal cruise altitude from Eq. (26).

Example

Assume the following vehicle characteristics:

$$W = 200{,}000 \text{ lb}$$
$$S = 2500 \text{ ft}^2$$
$$K = 0.05 = \text{const}$$
$$C_{D0} = 0.015 \qquad M \le 0.8$$
$$= 0.015 + (M - 0.8)^2 \qquad M \ge 0.8$$
$$c = 0.6 \text{ lb/h/lb thrust}$$

Best cruise speed or Mach number for a given altitude is obtained from the solution of Eq. (24)

$$-qSC_{D0} + 3\frac{KW^2}{qS} - qSMC'_{D0} = 0 \tag{31}$$

The solution for best cruise Mach number vs altitude is presented in Fig. 1. It can be observed that best cruise Mach number increases with increasing altitude. Best n.mi./lb, V/cD, is presented in Fig. 2. The drag as a function of best cruise Mach number is presented in Fig. 3. Minimum drag occurs at approximately 45,000 ft. Below approximately 30,000

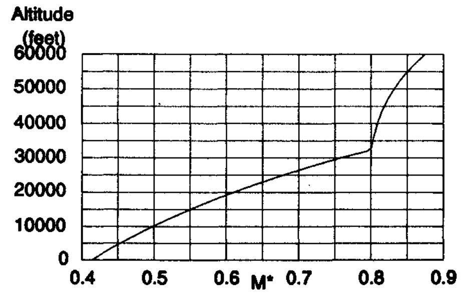

Fig. 1 Best cruise Mach number.

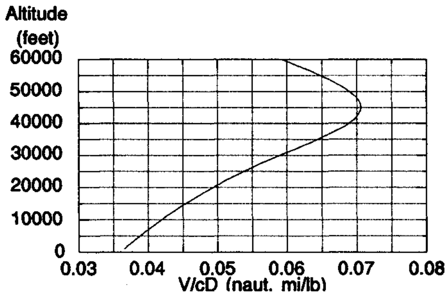

Fig. 2 Best cruise n.mi./lb.

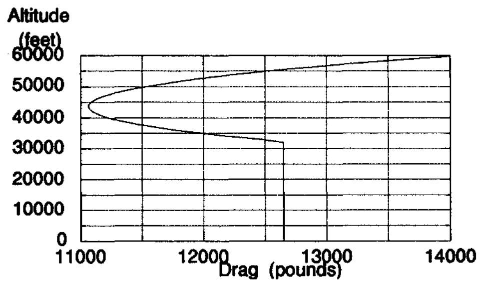

Fig. 3 Drag at best cruise Mach number.

ft where M^* is less than 0.8, q^* is constant. Thus the drag is constant.

The data in these figures does not include the possibility that the aircraft may be thrust limited. If limited, that would preclude the possibility of reaching the global optimum.

The analytical solution is obtained from Eq. (43)

$$M^* = 0.81875$$

The optimal value for the dynamic pressure is

$$q^* = 144.38 \text{ lb/ft}^2$$

$$\sigma = 0.19336$$

Optimal cruise altitude is approximately

$$h^* = 45{,}120 \text{ ft}$$

Conclusions

It has been demonstrated that best cruise speed is in the drag rise region if sufficient thrust is available. Best cruise altitude corresponds to the altitude where drag is a global minimum. These results disprove the theory that the cruise lift to drag ratio is

$$\frac{3^{1/2}}{2}\left(\frac{L}{D}\right)_{\max}$$

where max lift-to-drag ratio is based upon constant aerodynamic drag coefficients; that is prior to drag rise.

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Vortex Breakdown Studies of a Canard-Configured X-31A-Like Fighter Aircraft Model

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Introduction

THE use of canard configurations as a potential method for improved aerodynamic performance has received considerable attention recently, both experimentally and computationally. Since the experimental study by Behrbohm,[1] there have been a number of canard-related investigations.[2] The more recent experimental studies by Er-El and Seginer,[3] Calarese,[4] and Hummel and Oelker[2] focused on the interaction mechanism of canard and wing vortex systems. Considerable progress has also been made in the computation of high angle-of-attack (AOA) flows over delta and double-delta wings, canard-wing combinations,[5] and even complete aircraft configurations, such as the F-18 and X-31 aircraft.[6]

As pointed out in Ref. 2, the flow physics of the canard-wing configuration is still not sufficiently understood and documented. The high AOA flight is limited by the vortex breakdown phenomenon and by the onset of vortex asymmetry. The canards have a strong influence on the vortex development and on the lateral and directional stability. Of special importance is the understanding of the vortex development and breakdown (bursting) under rapidly maneuvering conditions as envisioned for the X-31A aircraft. Therefore, this investigation was undertaken to characterize the vortical flowfield around a maneuvering canard-configured fighter aircraft model comparable to X-31A. It included extensive static and dynamic flow visualization experiments in the 0–50-deg AOA range using dye-injection technique in the Naval Postgraduate School (NPS) water tunnel. Additional details of the investigation appear in Refs. 7 and 8.

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